Engineering Notes

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T-Burners: Experiments Compared With Theory

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Nomenclature

f = frequency of first longitudinal mode
k_1 = constant used in Eq. (1)
k_2 = constant used in Eq. (1)
M_b = Mach number of gases leaving burning surface
$\hat{p}$ = local amplitude of pressure oscillations
$\hat{p}_m$ = local amplitude of pressure oscillations at burner end
$\hat{p}_0$ = local amplitude of pressure oscillations at time zero
R_b = response function of burning propellant surface
S_b = total area of burning propellant in each end of burner
S_{bs} = additional area of burning propellant used in each end of variable-position T-Burner
S_c = cross-sectional area of burner
t = time
Z = distance of additional propellant samples from nearest burner end
Z_T = length of T-Burner
α = exponential growth rate of oscillations
α_0 = exponential growth rate of oscillations with reference amount of propellant
α_d = constant characterizing acoustic losses in burner

THROUGHOUT the history of solid propellant rockets, oscillatory combustion has been encountered frequently. Usually the undesirable consequences associated with oscillatory combustion have been largely overcome by expensive cut-and-try methods. The development of the T-Burner[1] has been directed toward avoiding this procedure. Early in the development of this tool, different propellants could be qualitatively compared and those less susceptible to oscillatory combustion selected for use. Subsequently, techniques improved to the point that propellants could be characterized quantitatively.[2]

A comprehensive theory of oscillatory combustion was developed more or less simultaneously with the experimental development by McClure, Hart, Cantrell, and others at the Johns Hopkins University Applied Physics Laboratory.[3-5] These investigators published their last paper in 1965. This paper presents general equations for the rate of growth of acoustic oscillations in both rocket motors and T-Burners. A very extensive effort was devoted to this development, the goal of which was to enable the designer to combine inexpensive T-Burner measurements with appropriate computations to predict the degree of linear stability that might be expected in a full-scale motor.

At the time of the publication of Ref. 5 it seemed that the desired goal was close at hand. It is true that the procedures were largely unproven and also that the nonlinear oscillatory amplitude could not be predicted with the linearized theory. Nevertheless, the success with which the analysis had been applied to T-Burners engendered optimism that problems associated with oscillatory combustion in large rocket motors could soon be avoided. As a result of recent studies, however, a controversy has arisen over both how T-Burner data should be interpreted and how it should be employed in motor design calculations. Consequently, it has become apparent that the goal of reliable techniques for solving oscillatory combustion problems in a rigorous quantitative manner is not as close as it had appeared in 1965.

Like all models, that of Hart and McClure contained several simplifications and assumptions. A recent study of the problem by Culick[6] concentrated on one of the areas of uncertainty and indicated the presence of a new source of acoustic damping. This damping of the oscillations is predicted to occur as the result of interactions between the acoustic and the mean flow fields.[6] Culick's ideas were later supported by a simplified analysis carried out by Coates.[7] In a later publication, however, Culick's equations were revised[8] to include vent effects which tended to offset the damping indicated in Ref. 6. More recently, Coates and Horton[9] have shown that depending on the assumptions one makes, the original McClure-Hart theory can result in a mean flow damping not present in the original work. The equations resulting from this analysis conflict with the most recent equations derived by Culick, however. The present work was undertaken to evaluate the relative merits of the two recent theories in correlating and explaining experimental data.

Analysis

For a detailed discussion of the theories, the reader is referred to Refs. 8 and 9. It is sufficient here to present the results along with such discussion as is required to compare the theories to the experiment.

It is possible to combine the two theoretical approaches[6,9] and write a general equation containing constants whose magnitude differs according to which of the controversial assumptions is used. If this is done and velocity coupling is neglected, the general instability equation for the T-Burners shown in Fig. 1a and 1b becomes

$$\alpha = 4fM_b\left\{\int_{S_b}\left[(R_b+k_1)\left(\frac{\hat{p}}{\hat{p}_m}\right)^2 - k_2\right]\frac{dS_b}{S_c}\right\} - \alpha_d \quad (1)$$

where the exponential growth rate of the oscillations is

$$\hat{p} = \hat{p}_0 \exp(\alpha t) \quad (2)$$

According to Culick, the values of k_1 and k_2 are 1 and 0, respectively, while Coates and Horton prefer values of 0 and 1.

Of interest also is the instability expected from the Quarter-Wave Burner shown in Fig. 1c which is predicted by both theories to be

$$\alpha = 4fM_b \int_{S_b}(R_b-1)(dS_b/S_c) - \alpha_d \quad (3)$$

Examination of these equations suggested two methods of discriminating between the different possible values of k_1 and k_2. One method involved testing the same propellant in both the Quarter-Wave and T-Burners. Then, a comparison of the two sets of results would suggest the best values for k_1 and k_2. A second method would involve obtaining data with propellant placed in various locations along the side of the burner shown in Fig. 1b. Equation (1) could then be used to analyze the data and values for k_1 and k_2 selected from a best-fit

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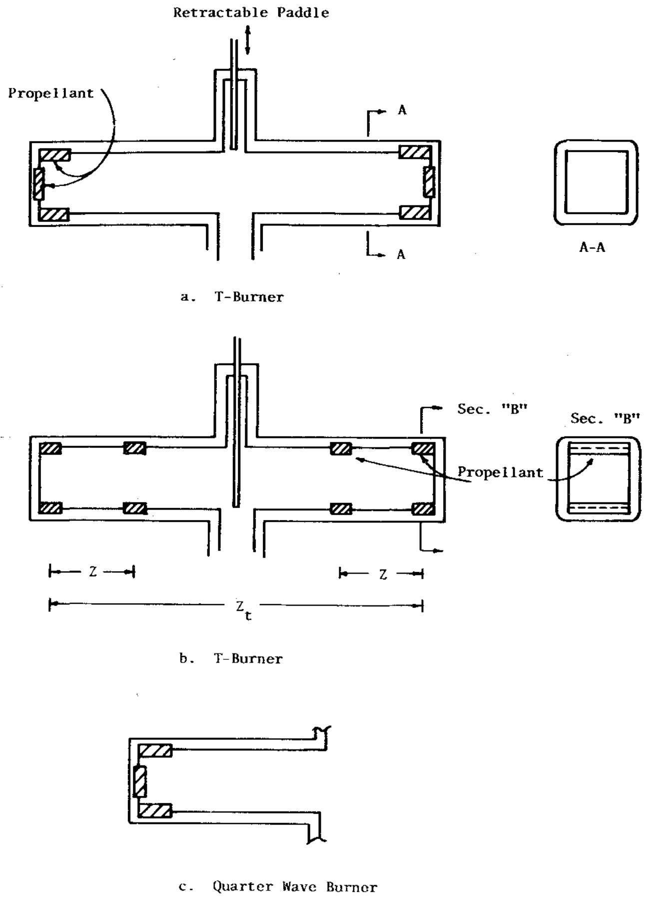

Fig. 1 Various burners used to test propellants.

criterion. Both of these methods were used to obtain data and the experiments are described in the following sections.

Burners

The object of the experimental program was to carry out T-Burner tests that would make it possible to select the best theoretical approach. Accordingly, tests were conducted with the three burner types shown in Fig. 1.

The first burner was a regular, double-ended T-Burner (Fig. 1a) that employed several unique features. First, it was smaller than most described in the literature, having a cross-sectional area of approximately one square inch. Second, the cross section was nearly square rather than circular. Third, there were recesses machined in the internal walls of the burner into which flat slab-shaped propellant samples could be placed. The burner was also equipped with a retractable air-driven paddle located opposite the vent. The propellant was ignited with the paddle inserted into the burner in which position it suppressed unstable burning. When steady burning was established and the propellant samples had burned to the point where the burning surface was very near flush with the tube walls, the paddle was rapidly ($\sim$20 m/sec) retracted and oscillations were permitted to grow.

A second burner used in the study is shown in Fig. 1b and differed from the one described above only in that additional propellant samples were placed at various positions between the ends and the vent. In different runs, the additional samples were placed at different positions and, thus, the instability was determined as a function of the sample position.

The Quarter-Wave Burner is depicted in Fig. 1c. This burner was constructed with the same square tube material and the same type of sample holders as were used in the regular T-Burner. To simulate an opening attached to an infinite flange, the open end of this burner was attached to a two-foot-square plate of steel. The metal plate had a hole in the center that matched the hole in the burner.

Tests

It was desired to test a propellant in the Quarter-Wave Burner and also in the T-Burner. Then the comparative stability could be determined. Propellant selection was based on a preliminary series of tests in which the qualitative instability of several propellants was determined. It was found that two propellants, A-18 and A-17, were unstable at atmospheric pressure and burner lengths corresponding to 600 Hz. Therefore, these propellants were used to carry out the program and the tests were conducted at atmospheric pressure. A-18 was the designation of an ammonium perchlorate oxidized composite with a polybutadiene-acrylonitrile binder. It contained 75% 15-μ oxidizer, 24% binder, and 1% LiF as a burning rate modifier. A-17 was identical except that the oxidizer was 80 μ in diameter. Although unusual, the testing at a mean pressure of one atmosphere provided the data necessary to evaluate the theories. The use of this pressure greatly simplified testing, although it sacrificed nothing since the questions to be answered are not dependent on mean pressure but only on mass flow rates.

In the first portion of the comparative tests, the variable-area technique[10] was used with the T-Burner of Fig. 1a. Propellant samples 1 in. wide, $\frac{1}{4}$ in. thick and of the desired length were machined and the sides and back inhibited. The samples were then placed in the recessed cavities at the ends of the T-Burner. If the propellant used for a particular test did not fill all of the sample cavities, $\frac{1}{8}$ in. thick metal fillers were used to fill the vacant portion and provide smooth walls. The $\frac{1}{4}$ in. thick propellant samples protruded into the chamber, of course, and the burning surface was not flush until the propellant was half consumed. The surfaces of the samples were painted with an igniter paste to promote rapid, uniform ignition. Ignition was accomplished by the use of hot wires coated with igniter paste located in each of the burners. The over-all length of the burner was 23 in. The amounts of propellant in each end of the burner ranged from about $\frac{1}{2}$ in.^{2} to 3.9 in.^{2} As indicated in Fig. 1a, the propellant samples could be mounted on both the end wall and the side walls. In all, 9 tests were conducted with A-18 propellant and 10 with A-17.

The other portion of the comparative tests was conducted with the Quarter-Wave Burner. The procedure was the same as that used with the T-Burner except that the suppression device required a different mounting arrangement. Useful data were obtained from 8 tests in which A-18 propellant was used. The propellant area ranged from 1.5 to 2.2 in.^{2} Several tests that utilized less than 1.5 in.^{2} of propellant were stable and otherwise yielded no useful information. Another 8 successful tests were conducted with A-17 propellant with the propellant area varying between 3.4 and 5.0 in.^{2}

Another group of tests, dubbed the variable position tests, were conducted in the T-Burner shown in Fig. 1b. The objective of these tests was to measure the effect of varying the sample position upon the growth constant of the oscillations. Then the measured effect could be compared to that predicted by the different theories.

The A-18 propellant samples were located at the ends of the burner and at one of four different positions along the length of the 24.2-in. long burner. Five different configurations were tested, with duplicate tests being made with each configuration. The first 2 tests were made with a single set of opposed samples having a burning area of 1 in.^{2} placed at each end of the burner. In the other four configurations, an additional set of opposed samples were located in each end of the burner. In the second configuration, the additional samples were placed immediately adjacent to those at the ends of the burner. In the third configuration, these additional samples were moved

Table 1 Summary of experimental data

Burner	Propellant location	S_b/S_c	Frequency h_z	Growth constant sec^{-1}
		A-18 propellant		
T	end	0.46	600	5
T	end	0.47	590	7
T	end	0.86	640	16
T	end	0.84	640	19
T	side	1.44	660	43
T	side	1.58	660	49
T	side	0.58	560	14
T	side	0.56	550	7
T	side	0.58	560	4
Quarter wave	side	1.53	570	3
Quarter wave	side	1.46	570	4
Quarter wave	side	2.28	590	31
Quarter wave	side	2.18	590	47
Quarter wave	side	2.24	590	65
Quarter wave	side	2.10	590	68
Quarter wave	side	2.11	580	54
Quarter wave	side	1.94	600	36
		A-17 propellant		
T	side	1.83	650	29
T	side	1.94	650	18
T	side	0.95	610	12
T	side	1.93	680	28
T	side	2.78	710	37
T	side	3.90	670	71
T	side	0.99	585	11
T	side	0.99	585	6
T	side	2.96	640	52
T	side	3.93	630	71
Quarter wave	side	3.94	650	10
Quarter wave	side	3.88	570	4
Quarter wave	side	4.78	570	28
Quarter wave	side	3.50	545	4
Quarter wave	side	4.86	570	23
Quarter wave	side	3.40	540	11
Quarter wave	side	4.88	520	26
Quarter wave	side	5.00	530	38
Variable position	end	1.84	670	31
Variable position	end	1.84	660	29
Variable position	end + Z/Z_T = 0.06	1.84 + 1.84	700	69
Variable position	end + Z/Z_T = 0.06	1.84 + 1.84	710	65
Variable position	end + Z/Z_T = 0.182	1.84 + 1.84	705	45
Variable position	end + Z/Z_T = 0.182	1.84 + 1.84	715	42
Variable position	end + Z/Z_T = 0.244	1.84 + 1.84	710	23
Variable position	end + Z/Z_T = 0.244	1.84 + 1.84	715	25
Variable position	end + Z/Z_T = 0.364	1.84 + 1.84	...	stable
Variable position	end + Z/Z_T = 0.364	1.84 + 1.84	...	stable

toward the vent a distance of 3.9 in. from the ends, and, in the fourth and fifth configuration, they were located 5.4 and 8.3 in., respectively, from the ends.

The first two tests that employed only a single pair of propellant samples in each end were used as reference runs. In the subsequent runs, burning an additional pair of samples in each end, the growth constants were greater than the reference runs when the additional samples were near the ends and less than the reference runs when they were nearer the vent. The burner was completely stabilized when these samples were closest to the vent. Thus, these samples could drive or dampen the pressure oscillations depending upon their location in the burner.

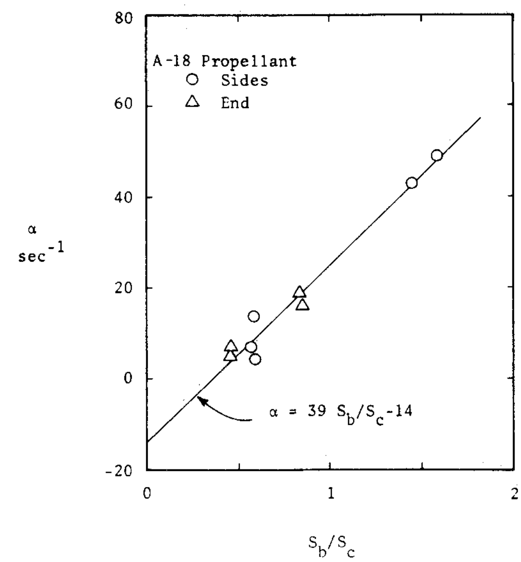

Fig. 3 Experimentally determined growth constants for A-18 propellant in the T-Burner.

Results and Discussion

The results of each of the 43 test firings consisted of a pressure-time history as measured at the closed ends inside the burners. During the portion of the pressure-time trace where the oscillations were growing, an exponential growth constant was measured. Table 1 gives these growth constants as well as the values of several other pertinent variables. Figures 2–4 were plotted from the data for the Quarter-Wave and T-Burners. Through the data were drawn the best-fit straight lines as determined by the least-squares method.

If the propellant samples are flush with the burner wall,

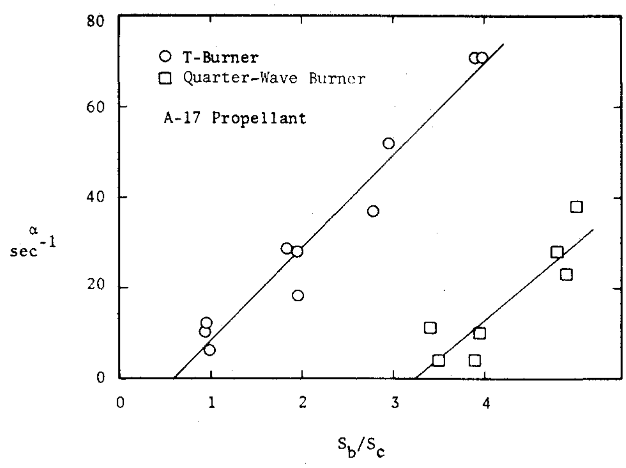

Fig. 2 Experimentally determined growth constants for A-17 propellant. Best-fit lines are $\alpha = 20.7\ S_b/S_c - 12.5$ for the T-Burner and $\alpha = 16.8\ S_b/S_c - 54$ for the Quarter-Wave Burner.

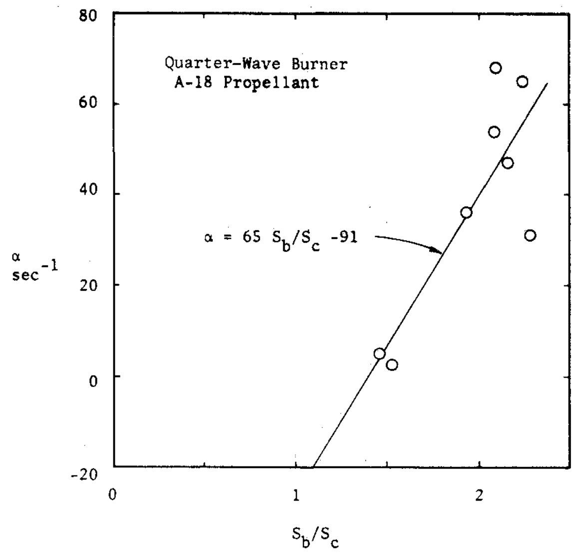

Fig. 4 Experimentally determined growth constants for A-18 in the Quarter-Wave Burner.

Table 2 Experimental response function values

Theory	$k_1 - k_2$	R_b A-17	A-18
Modified Hart-McClure			
T-Burner	−1	1.6	2.1
Quarter Wave Burner	−1	1.5	2.9
Culick ($k_1 = 1, k_2 = 0$)			
T-Burner	1	−0.4	0.1
Quarter-Wave Burner	−1	1.5	2.9

mounted near the end, and have lengths that are small, relative to the burner length, Eqs. (1) and (3) can be simplified to

$$\alpha = 4fM_b[R_b + k_1 - k_2](S_b/S_c) - \alpha_d \tag{4}$$

where k_1 and k_2 have the following values: original McClure-Hart-Cantrell Theory, $k_1 = 0$, $k_2 = 0$; Coates-Horton Modification of MHC Theory, $k_1 = 0$, $k_2 = 1$; Culick Theory, $k_1 = 1$, $k_2 = 0$ T-Burner, $k_1 = 0$, $k_2 = 1$ Quarter-Wave Burner, and where α_v represents the damping attributable to particles in the gas, wall friction, heat loss, and acoustic radiation from the vent.†

Equation (4) was employed to compute response functions according to the different theories from the least-squares, best-fit slopes of the lines shown in Figs. 2–4. The computed response functions are tabulated in Table 2. It is apparent that the MHC theory as modified by Coates and Horton yields more consistent response function values from the two different burner configurations.

To analyze the data from the variable position T-Burner, Eq. (2) was simplified through the use of the fact that the propellant samples were short relative to the burner length. Additionally, it was assumed that the spatial distribution of the pressure wave could be described by a cosine function. Then the incremental growth constant caused by the additional propellant samples is

$$\alpha - \alpha_0 = 4fM_5[(R_b + k_1)(\hat{p}/\hat{p}_m)^2 - k_2](S_{bs}/S_c) \tag{5}$$

where α_0 denotes the average measured growth constant for the reference runs where propellant was located in the ends only and (S_{bs}/S_c) characterized the additional propellant used in the other runs.

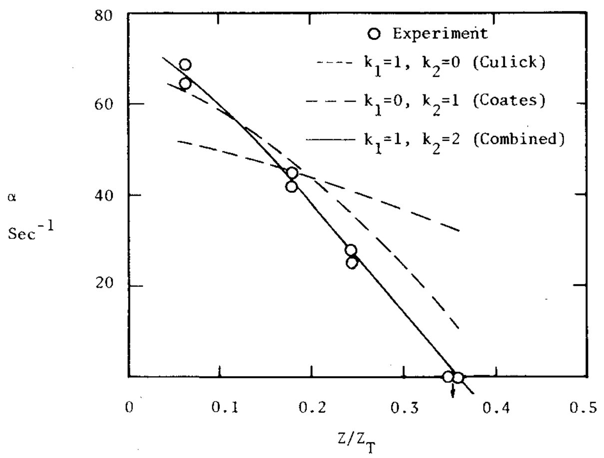

Fig. 5 Comparison of analysis and experimentally determined growth constants for A-18 propellant in variable position T-Burner.

† Vent radiation is expected to be negligible for the T-Burner, but rather large for the Quarter-Wave Burner, being given approximately by the equation $\alpha_{rad} = 4f(\pi^2/8)(D/L)^2$.

Table 3. Statistical comparison of theory and data

Theory		R_b	$\Sigma(\alpha - \alpha_i)^{2a}$
$k_1 = 0$	$k_2 = 1$	2.46	831 sec^{-2}
$k_1 = 1$	$k_2 = 0$	−0.02	3201 sec^{-2}
$k_1 = 1$	$k_2 = 2$	2.64	25 sec^{-2}

[a] $\Sigma(\alpha - \alpha_i)^2$ is the sum of the squares of the deviations between the best-fit theoretical line and the individual data points.

Figure 5 shows the data from the variable position tests plotted as α vs Z/Z_t, where Z represents the average distance of the variable position samples from the ends of the burner, and Z_t is the burner length. The least-squares best-fit values of R_b for the data were determined for the different theories and these are listed in Table 3. The combination of $R_b = 2.64$, $k_1 = 1$, and $k_2 = 2$ resulted in an amazingly good correlation of the data, substantially better than the other combinations that were tried and better than could be expected from the precision of the data. A re-examination of Eq. (4) and the results shown in Table 2 reveals that the same values ($k_1 = 1$, $k_2 = 2$) yields identical results to those with $k_1 = 0$, $k_2 = 1$. Thus, all of the data can be satisfactorily correlated by constant values of $k_1 = 1$ and $k_2 = 2$.

Although there has not been a published analysis that presents this combination of values, they will result if reasonable postulates are used along with either existing analytical framework. The value of $k_1 = 1$ results from the assumption that propellant gases entering the chamber from the side achieve the local acoustic velocity irreversibly. The value of $k_2 = 2$ results if the assumption is added to the aforementioned that the gases leaving the chamber convect the local acoustic energy with them. Both of these assumptions are at least plausible.

In summary it is seen that two groups of experimental data are consistently explained by existing analysis provided that appropriate assumptions are made. It seems, then, that T-Burner data are best reduced by the use of these assumptions, namely that there are both flow-turning and vent losses in the T-Burner.

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